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Appendix B

Derivation of the Regional Type I Error Rate. Let Z_1 and Z_{1c} be the test statistics for the first region and regions other than the first region, respectively.

$$Z_1 = (\overline{X}_{1.} - \overline{Y}_{1.}) / (\sigma\sqrt{2/n_1})$$

$$Z_{1c} = \sum_{i=2}^{K}\sum_{j=1}^{n_i}\left(X_{ij} - Y_{ij}\right)\Big/\left(\sigma\sqrt{2/(N-n_1)}\right).$$

Additionally, Let, D_{1c} be the observed mean difference for regions other than the first region. Hence,

$$D_{1c} = \sum_{i=2}^{K}\sum_{j=1}^{n_i}\left(X_{ij} - Y_{ij}\right)\Big/\left(N-n_1\right).$$

We can then derive the mathematical formula for the regional type I error (α_s), as shown below.

$$\alpha_s = P_\delta\left(D_1 \ge \rho D | Z > z_{1-\alpha}, H_{s0} : \Delta_1 \le \rho\Delta\right)$$

$$= P_\delta\left(D_1 \ge \rho(p_1 D_1 + (1-p_1)D_{1c} | Z > z_{1-\alpha}, H_{s0}\right)$$

$$= P_\delta\left(Z_1 > \frac{\tilde{n}\sqrt{p_1(1-p_1)}}{1-\rho p_1} Z_{1c} | \sqrt{p_1}Z_1 + \sqrt{1-p_1}Z_{1c} > z_{1-\alpha}, H_{s0}\right)$$

$$= P_\delta(Z_1 - \frac{\Delta_1}{\sigma\sqrt{\frac{2}{n_1}}} > \frac{\rho\sqrt{p_1(1-p_1)}}{1-\rho p_1}\left(Z_{1c} - \frac{\Delta_{1c}}{\sigma\sqrt{\frac{2}{N-n_1}}}\right) + \frac{\rho\sqrt{p_1(1-p_1)}}{1-\rho p_1}\frac{\Delta_{1c}}{\sigma\sqrt{\frac{2}{N-n_1}}}$$

$$= P_\delta(Z_1 - \frac{\Delta_1}{\sigma\sqrt{\frac{2}{n_1}}} > \frac{\rho\sqrt{p_1(1-p_1)}}{1-\rho p_1}\left(Z_{1c} - \frac{\Delta_{1c}}{\sigma\sqrt{\frac{2}{N-n_1}}}\right) + \frac{\rho\sqrt{p_1(1-p_1)}}{1-\rho p_1}\frac{\Delta_{1c}}{\sigma\sqrt{\frac{2}{N-n_1}}}$$

$$-\frac{\Delta_1}{\sigma\sqrt{\frac{2}{n_1}}} | \sqrt{p_1}\left(Z_1 - \frac{\Delta_1}{\sigma\sqrt{\frac{2}{n_1}}}\right) + \sqrt{1-p_1}\left(Z_{1c} - \frac{\Delta_{1c}}{\sigma\sqrt{\frac{2}{N-n_1}}}\right) > z_{1-\alpha} - \sqrt{p_1}\frac{\Delta_1}{\sigma\sqrt{\frac{2}{n_1}}} - \sqrt{1-p_1}\frac{\Delta_{1c}}{\sigma\sqrt{\frac{2}{N-n_1}}}, H_{s0}$$

$$= P_0\left(\begin{array}{l} Z_1^* \ge \frac{\rho\sqrt{p_1(1-p_1)}}{1-\rho p_1} Z_{1c}^* + \frac{\rho\sqrt{p_1(1-p_1)}}{1-\rho p_1}\frac{\Delta_{1c}}{\sigma\sqrt{\frac{2}{N-n_1}}} - \frac{\Delta_1}{\sigma\sqrt{\frac{2}{n_1}}} | \sqrt{p_1}Z_1^* + \\ \sqrt{1-p_1}Z_{1c}^* > z_{1-\alpha} - \frac{1}{\sigma\sqrt{\frac{2}{N}}}\left(p_1\Delta_1 + (1-p_1)\Delta_{1c}\right) \end{array}\right)$$

$$= \frac{P_0(Z_1^* > d_1 Z_{1c}^* + d_2, d_3 Z_1^* + d_4 Z_{1c}^* > d_5)}{P_0(d_3 Z_1^* + d_4 Z_{1c}^* > d_5)}$$

$$= \frac{\int_{b_1}^{\infty} \left(\Phi\left(\frac{1}{d_1} u - \frac{d_2}{d_1} \right) - \Phi\left(-\frac{d_3}{d_4} + -\frac{d_5}{d_4} \right) \right) \phi(u) du}{\int_{-\infty}^{\infty} \left[1 - \Phi\left(-\frac{d_3}{d_4} + -\frac{d_5}{d_4} \right) \right] \phi(u) du}$$

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Where,

$$d_1 = \rho \frac{\sqrt{p_1(1-p_1)}}{1-\rho p_1}, \quad d_2 = d_1 \frac{\Delta_{1c}}{\sigma\sqrt{\frac{2}{N-n_1}}} - \frac{\Delta_1}{\sigma\sqrt{\frac{2}{n_1}}}, \quad d_3 = \sqrt{p_1}, \quad d_4 = \sqrt{1-p_1}$$

$$d_5 = z_{1-\alpha} - \frac{1}{\sigma\sqrt{\frac{2}{N}}} \left(p_1 \Delta_1 + (1-p_1) \Delta_{1c} \right), \; b_1 = d_2 + \frac{d_1 (d_5 - d_2 c_3)}{d_1 d_3 + d_4}$$